POF 291 FLUTTER VELOCITY RESULTS PRODUCES NEGATIVE MARGINS OF SAFETY COMPARED TO NACA TN 4197

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The author of POF 291 claims his equations are based on results from NACA TN 4197. The author of POF 291 also erroneously claims to have included "**a more accurate term for torsional modulus, G**" (or did the author mean, J), by the "**inclusion of plate theory"** a modification not included in the well documented and validated original NACA paper. In addition, the author stated he provided no details about the modification because of the "**complex nature of the flutter boundary equation**". A comparative analysis in Figure-1 illustrates the difference between the original NACA flutter velocity and the POF 291 flutter velocity is the value, 2 under the SQRT sign of the POF 291 flutter equation. Therefore, POF 291 generates a negative margin of safety for flutter velocity causing flutter velocity to be overestimated by 41.4% compared to the NACA study.

According to NACA TN 4197 the effective shear modulus, G_E for solid fins always equals the uniform material shear modulus, G. This relationship can be verified after substituting torsional modulus, J for a solid thin airfoil (Equation-10) into the equation for effective shear modulus, G_E (Equation-12) for fabricated fins and then deriving the relationship, G_E = G for solid fins. According to NACA TN 4197, G_E always equals G for fins having a solid cross-section as demonstrated by the discussion on page 13. Also, according to NACA TN 4197, G_E greater than G may only be achieved for airfoil sections stiffened by bulkheads or by using a material with a greater shear modulus, G. This discussion proves that for a solid fin, G_E = G not G_E = 2G as used in POF 291 which increases flutter velocity by factor, 1.414.

The error discovered in POF 291 causes flutter velocity to be overestimated by 41.4%. To discover the source of the POF 291 error a detailed mathematical analysis of every equation in NACA TN 4197 using Mathcad was performed. This deep dive into the NACA TN 4197 flutter equations revealed the source of the error as a modified **section torsional modulus, J** defined as Equation-10. POF 291 multiplied Equation-10 by the factor 2 then substituted the modified version of Equation-10 into Equation-12 the **effective shear modulus**, $\mathbf{G_E}$ of an equivalent section for fabricated wings. This modification increased the material property, G for a solid fin by a factor of 2 meaning G_E = 2G and not G_E = G as intended by the authors of NACA TN 4197 for solid wings and fins. The authors of NACA TN 4197 intended the shear modulus, G_E to be equal to G for solid uniform wing and fin sections. The NACA paper explicitly demonstrates that G_E = G for solid fin sections by substituting Equation-10 into Equation-12. The modified version of Equation-10 for J is the value presented in undergraduate solid mechanics textbooks not plate theory. However, the form for Equation-10 is irrelevant because the authors of NACA TN 4197 only proposed using Equation-12 to measure effective shear modulus, G_E for fabricated wings with bulkheads by performing twist tests in the laboratory. As further proof that G_E = G for solid fin sections please consult the textbook “Aeroelasticity” by Bisplinghoff, Ashley and Halfmann and reference 6 in NACA TN 4197 by Theodorsen and Garrick. But, as the NACA paper demonstrates, G_E = G for solid fin sections and not G_E = 2G as used in POF 291 which increases flutter velocity by 41.4% and generates a **negative margin of safety** of the same amount. As a side note, unmanned flight vehicles normally require a margin of safety of at least 25%. The main question that was answered during this validation was the exact form of the section torsional modulus, J. It was determined that after inserting $c = c_{root}/2$ into the classical equation for section torsional modulus, $J = \frac{ct^3}{3}$ the resulting equation, $J \approx \frac{ct^3}{6}$ is the approximate equation for **section torsional modulus, J** displayed in NACA TN 4197 as Equation-10. However, this equation for J is not used to compute flutter velocity other than in Equation-12 to determine G_E for fabricated wings only.

FIN GEOMETRY INPUT VALUES

Fin root chord length	$c_r := 9.75 \cdot in$
Fin Tip chord length	$c_t := 3.75 \cdot in$
Fin constant thickness	$t := 0.125 \cdot in$
Fin semi-span root to tip	$b := 4.75 \cdot in$
Shear modulus of elasticity	$G := 380000 \cdot psi$

VALUES FOR THE FLUTTER VELOCITY ANALYSIS

Fin surface area

$S := \frac{1}{2} \cdot (c_r + c_t) \cdot b$ $\quad S = 32.06 \cdot in^2$

Fin aspect ratio

$AR := \frac{b^2}{S}$ $\quad AR = 0.704$

Ratio of fin tip chord to fin root chord

$\lambda := \frac{c_t}{c_r}$ $\quad \lambda = 0.385$

NASA ATMOSPHERIC MODEL, h < 36152 feet (Troposphere)

Sea level atmospheric pressure	$P_0 := 14.69 \cdot psi$
Altitude for maximum velocity	$h := 3000$

$T := 59 - .00356 \cdot h$ $\quad T = 48.32$

$P := 2116 \cdot \frac{lbf}{ft^2} \cdot \left(\frac{T + 459.7}{518.6}\right)^{5.256}$ $\quad P = 13.19 \cdot psi$

$a := \sqrt{1.4 \cdot 1716.59 \cdot (T + 460)} \cdot \frac{ft}{sec}$ $\quad a = 1105.26 \cdot \frac{ft}{sec}$

NACA TN 4197 Flutter Velocity Equation used in FinSim

$$U_F := \sqrt{\frac{\frac{G}{psi}}{\left[\frac{39.3 \cdot (AR)^3}{\left(\frac{t}{c_r}\right)^3 \cdot (AR + 2)} \cdot \left(\frac{\lambda + 1}{2}\right) \cdot \left(\frac{P}{P_0}\right)\right]}} \cdot a$$

$U_F = 380.095 \cdot mph$ $\quad U_F = 557.473 \cdot \frac{ft}{sec}$

$M_F := \frac{U_F}{a}$ $\quad M_F = 0.50438$

Original POF 291 Flutter Velocity Equation, From POF 291

$$U2_F := a \cdot \sqrt{\frac{G}{\frac{1.337 \cdot (AR)^3 \cdot P \cdot (\lambda + 1)}{2 \cdot (AR + 2) \cdot \left(\frac{t}{c_r}\right)^3}}}$$

$U2_F = 537.666 \cdot mph$ $\quad U2_F = 788.58 \cdot \frac{ft}{sec}$

$M2_F := \frac{U2_F}{a}$ $\quad M2_F = 0.71347$

POF 291 Flutter Velocity Equation rearranged to show new value for "G", Torsional Modulus

$$U3_F := a \cdot \sqrt{\frac{2 \cdot G}{\frac{1.337 \cdot (AR)^3 \cdot P \cdot (\lambda + 1)}{(AR + 2) \cdot \left(\frac{t}{c_r}\right)^3}}}$$

$U3_F = 537.666 \cdot mph$ $\quad U3_F = 788.58 \cdot \frac{ft}{sec}$

$M3_F := \frac{U3_F}{a}$ $\quad M2_F = 0.71347$

POF 291 "new" magical value for "G", shear modulus is G = 2G

$\sqrt{2} \cdot U_F = 537.53618 \cdot mph$ $\quad \sqrt{2} = 1.414$ $\quad \frac{U2_F}{U_F} = 1.415$

POF 291 Flutter Velocity Equation results, without modified "G" shown equal to NACA 4197

$$U4_F := a \cdot \sqrt{\frac{G}{\frac{1.337 \cdot (AR)^3 \cdot P \cdot (\lambda + 1)}{(AR + 2) \cdot \left(\frac{t}{c_r}\right)^3}}}$$

$U4_F = 380.187 \cdot mph$ $\quad U4_F = 557.61 \cdot \frac{ft}{sec}$

$M4_F := \frac{U4_F}{a}$ $\quad M4_F = 0.5045$

Figure-1, Mathcad results