ROCKET SPIN STABILIZATION USING CANTED FINS

FINSIM ANALYSIS
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Rotary Speed from Blade Element Theory

In Fin Blade-Element Theory the pitch, **P** is defined as the distance traveled by the rocket in one revolution of the fins. For an unyielding and incompressible medium the pitch distance **P** is equal to **Vrocket / Nx**, where the rotary speed, **Nx** = 1 rev/sec and **Vrocket** is the speed of advance of the rocket into the air. Therefore, the pitch angle or the angle between the rotational velocity and the upward rocket velocity is

Tan (Pitch Angle) = P / (2 π Rfin) or **Pitch Angle = atan [P / (2 π Rfin)]**

Where **Pitch Angle = 90 deg Fin Inclination Angle, Rfin** is the centerline radius to the tip of the fin and **Nx** is the rotation speed (rev/sec) of the rocket. Generally, the rocket will advance a total distance equal to **P Nx T** in **T** seconds of flight.

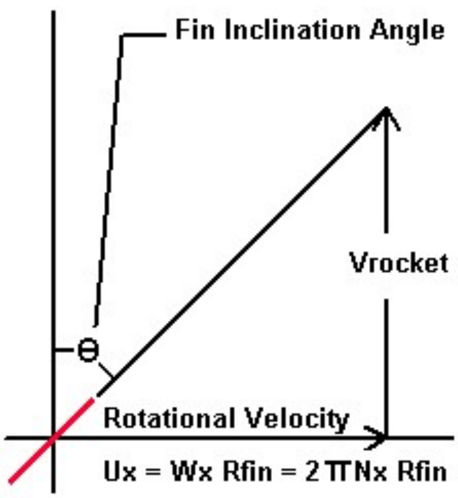


Figure 1, The rotational velocity, **Ux** is

Ux = Wx Rfin = 2 π Nx Rfin

Figure 1, the speed of rotation is

Nx = Vrocket / [2 π Rfin Tan (Pitch Angle)]

Wx = Vrocket / [Rfin Tan (Pitch Angle)]

In a real fluid such as air, there will be a certain amount of aerodynamic yielding or slip where the fin will not advance as far as predicted in one revolution of the rocket. However, for most fin designs the slip angle or the amount the pitch angle should be reduced is only about 3 percent. The assumption of an unyielding medium and the omission of slip angle will not introduce significant error for model rockets.

Forces Acting During Spin Stabilization

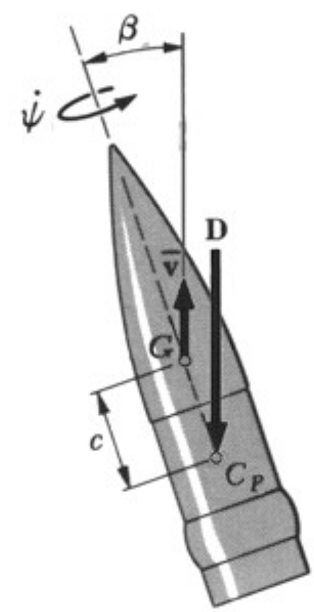


Figure 2 illustrates a spinning projectile about its axis of symmetry. The center of gravity (**G**) is the origin of an inertial reference frame. The following discussion defines the forces and moments acting on the projectile from classical mechanics.

At the instant shown in figure 2, the projectile has an angular velocity, **Wx** about its own axis. It also has an angular momentum **L** about this same axis, the axis making an angle β with the vertical. Two forces act on this representation of a projectile in flight. An upward force on the pivot at **G** and a force that acts downward at the center of pressure, **Cp**. The downward force acting at **Cp** is the total drag force acting on the rocket. The drag force is a function of velocity **V,** rocket shape as defined by the drag coefficient, **Cd** and air density**.**

In aerodynamic terms the drag force acting on the rocket is defined to be.
D = Cd * 1/2 * ρ-air * V^2 * Sref.

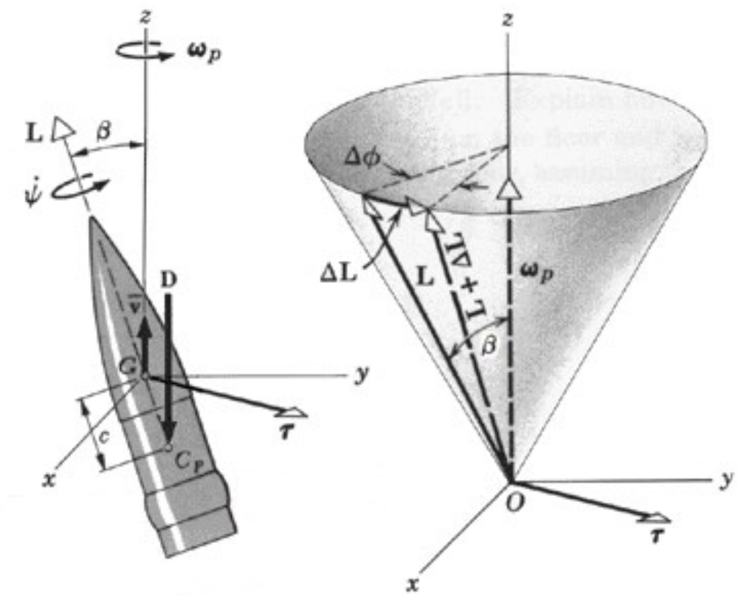
The drag force exerts a torque, **M** about the center of gravity, **G** given by
M = c X D = c D sin (β)

The vector **c** locates the center of pressure, **Cp** with respect to the center of gravity, **G** or pivot point of the rocket and is the origin of the inertial frame of reference.

Spin Stabilization and Angular Momentum

An external aerodynamic force changes the rocket's angular momentum **L** in time **dt** by an amount, **dL.** During this time interval the aerodynamic forces applied at the center of pressure, exert an average restoring torque given as **M** = **dL** / dt around the center of gravity.

The magnitude of this restoring torque is. **M = c D sin (β)**



Aerodynamic Coefficients

The moment coefficient and determination of the effective shift in the center of pressure location due to spin stabilization uses basic aerodynamic methods. The spin stabilization moment coefficient is defined as.
Cm = M / q Sref c

Where, **q** = **1/2 * ρ-air * V^2** is the dynamic pressure, **Sref** is the reference area based on the body diameter of the rocket and **c** is the static margin (XCp-Xcg) that acts as the reference moment arm-length.

Added stability of spin stabilization is quantified as the effective increase in the location of the center of pressure from the tip of the nose cone.
dXcp = Cm / Cn

Where, **Cn = CN_α * AOA** and is derived from a flight simulation program like RockSim or AeroCFD. **CN_α** is computed by RockSim or AeroCFD.

Finally, the effective **Cp** location of the spin stabilized rocket is.
Xcp_effective = dXcp + Xcp_nospin

Precession Rate and Precession Angle of Spin Stabilized Rocket

From experience we know that the axis of a rapidly spinning top or projectile will move around the vertical axis, sweeping out a cone. This motion is called precession. The rate of precession and the angle between the precession axis and the axis of symmetry is determined by gyroscopic effects. The rate of precession is computed from knowledge of the external moment acting on the projectile.

The external moment acting the projectile is.

$\mathbf{M = c \times D}$

The external moment or torque applied to the system is also equal to the time rate of change of the angular momentum, $\mathbf{dL}/dt$.

The following is derived from figure 3.

$\mathbf{dL = L\ d\varphi \sin(\beta)}$ implies $\mathbf{d\varphi = dL / (L \sin(\beta)}$

The rate of precession is.

$\mathbf{Wp = d\varphi / dt = (dL / dt)\ 1/(L \sin(\beta)}$

Where $\mathbf{d\varphi}$ is the angle between $\mathbf{L}$ and $\mathbf{L + dL}$ as the axis of symmetry sweeps around the precession axis.

Further:

Because $\mathbf{dL / dt = c \times D = c\ D \sin(\beta)}$ and $\mathbf{L = Ixx\ Wx}$ then

$\mathbf{Wp = c\ D / (Ixx\ Wx)}$

$\mathbf{Np = Wp / (2\pi)}$

Finally, without proof the precession angle is.

$\mathbf{\varphi = acos[(Ixx / Izz)(Wx / Wp)]}$

Where,

Ixx = Radial moment of inertia around symmetrical axis of rocket (kg-m^2, slug-ft^2).

Izz = Longitudinal moment of inertia around axis perpendicular to symmetrical axis.

Wx = Rotary frequency of rotation (radians/sec).

Nx = Rotary frequency of rotation (rev/sec) **Wp** =
Precession frequency of rotation (radians/sec).

Np = Precession frequency of rotation (rev/sec).

φ = Precession angle, angle between precession axis and axis of symmetry (degrees). In general $\mathbf{N = W / (2\pi)}$ to derive rate in rev/sec from rate in radians/sec.

References

Pages 260 to 263, 295 to 300 Physics, Halliday and Resnick

Pages 722 to 727, Dynamics, Beer and Johnson

MathCAD Spin Stabilization Analysis

Pitch angle of fins

$$\phi_{fin} := \frac{\pi}{2} - \theta_{fin}$$

Rotary speed of spin-stabilized rocket due to fin inclination

$$\omega_{x_i} := \frac{V_{rocket_i}}{r_{fin} \cdot \tan(\phi_{fin})} \qquad n_{x_i} := \frac{\omega_{x_i}}{2 \cdot \pi}$$

Drag of a spin-stabilized rocket

$$r := Xcp - Xcg$$

$$S_{ref} := \pi \cdot \frac{D_{rocket}^{2}}{4}$$

$$DRAG_i := \frac{1}{2} \cdot \rho_{air} \cdot \left(V_{rocket_i}\right)^2 \cdot S_{ref} \cdot Cd0$$

Frequency of precession Rate of precession

$$\omega_{p_i} := \frac{r \cdot DRAG_i}{Ixx \cdot \omega_{x_i}} \qquad n_{p_i} := \frac{\omega_{p_i}}{2 \cdot \pi}$$

Precession angle due to spin stabilization

$$\phi_i := \operatorname{acos}\left(\frac{Ixx}{Izz} \cdot \frac{\omega_{x_i}}{\omega_{p_i}}\right)$$

Moment exerted by precession of rocket drag around support (center of gravity)

$$M_{spin_i} := r \cdot DRAG_i \cdot \sin(\alpha)$$

Restoring pitch moment coefficient for rocket without spin stabilization

$$CN := CN_{\alpha} \cdot \alpha \qquad CM1_i := CN \cdot \left(\frac{Xcp}{L_{rocket}}\right)$$

Pitch moment coefficient due to spin stabilization

$$CMspin_i := \frac{M_{spin_i}}{\frac{1}{2} \cdot \rho_{air} \cdot \left(V_{rocket_i}\right)^2 \cdot S_{ref} \cdot r}$$

Rocket Cp location without spin-stabilization

$$Xcp_i := \frac{Xcp}{L_{rocket}}$$

Effective Cp location with spin-stabilization

$$Xcp_{spin_i} := \frac{CMspin_i}{CN} + Xcp_i$$

Spin Stabilization Instructions Step-By-Step Procedure

1. Develop your rocket model, load the rocket motor and perform a flight simulation as usual in RockSim. Actually, loading the rocket motor is necessary to allow RockSim to compute the center of gravity location (Cg) and to determine other mass effects required by the spin stabilization analysis.

2. Click on **Edit**, **Application Settings**, and then **Units** while in the main window of RockSim. Then, click **Use Default English Units** to put your RockSim model in the units required by FinSim.

3. This next step is the most difficult part of generating the required data for performing a spin stabilization analysis using RockSim and FinSim. This next series of steps is required to import the rocket velocity (V), drag coefficient (Cd), center of gravity location (Cg), center of pressure location (Cp), radial moment of inertia (Ixx), longitudinal moment of inertia (Izz) and normal force slope coefficient (CNa) of your rocket while in powered flight. A brief discussion of moment of inertia is necessary to give the user a feel for its importance in gyroscopic motion. Inertia forces are derived from the tendency of a mass to resist acceleration. The rotational mass of all rotating objects are represented by the mass moment of inertia term, I. Mass moment of inertia describes an object s resistance to rotational acceleration. Mass moments of inertia are computed by summing the products of all mass elements and the square of their distance from some reference axis location. In this case the origin of the inertial frame of reference is the center of gravity (Cg) location. Moment of inertia has units of mass-length^2 and can be kg-m^2, slug-ft^2, or ounces-in^2 as used in FinSim. For a spin stabilization analysis, the mass moment of inertia about the roll axis (Ixx) or radial mass moment of inertia and the mass moment of inertia about the pitch axis (Izz) or longitudinal mass moment of inertia must be determined to compute the resistance to external aerodynamic moment. We are lucky to have RockSim and its unique ability to compute Ixx and Izz because these terms are very difficult to measure experimentally using a torsional balance. The author knows by experience gained during the design of the Spint ABM cone rocket, that experimentally determining mass moment of inertia using a torsional balance is difficult and prone to experimental error.

4. The procedure to determine radial mass moment of inertia (Ixx), longitudinal mass moment of inertia (Izz) and the other four variables begins by clicking **Simulation** on the main toolbar in RockSim. Next, click **Export** and then select *y-velocity*, *Cd, CG, CP*, *Longitudinal mass moment of inertia, Radial mass moment of inertia,* and *CNa normal force coefficient* in the long list of variables that appear. Make sure all seven variables are highlighted or they will not be printed to the file you are about to export. To the right of **File Name:** type the name you want to call the exported values of moment of inertia, etc. Also, you can use **Browse** to put the *data file* into the AeroElastic folder, or the location most convenient when operating FinSim. The exported file contains V, Cd, CG, CP, Ixx, Izz, and CNa as a function of time from liftoff to parachute ejection. FinSim will read this data and average all the values for automatic entry into the data list. However, all variables are used as a function of

velocity within SpinSim. At this point you have accumulated all the most difficult information required to perform a spin stabilization analysis.

5. Run FinSim. From the main FinSim screen import the model s XML file. You need to import the RockSim XML file to define the fin geometry required to determine fin loads acting on the rocket. External loads are computed knowing the basic drag of the model (Cd) and the incremental drag of the fins canted to produce rotation as the model rockets into the atmosphere.

6. From the main tool bar in FinSim click on the SpinSim icon. Then on the toolbar in SpinSim click **File** and then click **Import RockSim Simulation Data**. Click on the CSV file you created in RockSim and exported. FinSim will automatically import Velocity, Cd, CG, CP, radial moment of inertia, longitudinal moment of inertia, and CNa and insert all six arrays into SpinSim. Ixx and Izz should have units, ounces-inches^2 as exported by RockSim into the CSV file. The other variables will have units of inches or no units if coefficients like Cd and CNa.

SpinSim Data Entered Manually

Now proceed to enter the remaining RockSim data values into the data list as follows.

a) Fin inclination angle: Fin inclination from the centerline of the rocket. This angle provides the rotation that gyroscopically stabilizes the rocket during flight.
b) Rocket AOA: This is the slight angle of attack that provides the external aerodynamic moment required to stabilize the rocket. The effect of spin stabilization is constant over a wide AOA, so values of 1 to 5 degrees AOA is sufficient for this analysis.
c) Radius to tip of fins: Distance from the center of the rocket to the very tip of the fins.
e) Total rocket length: This length represents the total length of the rocket.
d) Rocket reference diameter: This length represents the maximum diameter of the rocket and is used to compute the reference area for the analysis.

RockSim Simulation Data (Entered During Step 5)

e) Average rocket velocity. Velocity array inserted in step 5.
f) Radial moment of inertia: Ixx array inserted in step 5.
g) Longitudinal moment of inertia: Izz array inserted in step 5.
h) Cp location from nose tip: Cp location array inserted in step 5.
i) Cg location from nose tip: Cg location array inserted in step 5.
j) Total rocket Cd without spin: Cd array inserted in step 5.
k) Total rocket CNa: CNa array inserted in step 5.

7) Results: Click on **Plot selections**, then click the option button for **Cp location (Xcp) with spin stabilization** to determine the Cp location due to spin stabilization. Click on the remaining option buttons to display other important output information. Drag the slider bar to change velocity and display each variable as a function of rocket velocity.